Earlier this year, to assist one of our customers, Don Bently authored a short synopsis of work done by himself and Dr. Agnes Muszynska related to the defining equations for rotor dynamics. We felt the summary would be valuable for a broader audience and provide it here for the benefit of all our ORBIT readers.

Summary of Several Deficiencies in Classical Rotor Dynamic Theory, and Work Done by Bently Rotor Dynamic Research Corporation to Correct These Deficiencies

By Donald E. Bently

In some rotor dynamic studies published by the Electric Power Research Institute (EPRI) about 15 years ago, only a single degree of freedom was used.

The problem with a study based upon a single degree of freedom is that it is impossible to distinguish forward from reverse Dynamic Stiffness characteristics – both *x* and *y* directions must be used instead. Generally, the inability to distinguish between forward and reverse is a serious mistake because backward (reverse) stiffness introduces distinctly different effects than forward stiffness, and it is vital to differentiate between the two.

Bently Nevada, therefore, relies upon rotor dynamic studies that excite a rotor in both lateral directions, and measure its response in both lateral directions. To do this, we employ apparatus for circular perturbation, such as a rotating unbalance device, rather than a unidirectional device such as a shaker. Such studies allow us to separate the forward stiffness characteristic from the backward characteristic, and eliminate uncertainty and guesswork.

In addition to reliance upon experiments using two degrees of freedom, Dr. Agnes Muszynska and I have introduced three major changes to the fundamental rotor dynamic equations.

1. Cross Stiffness Term

Much existing literature uses so-called “cross stiffness” nomenclature. Further, the cross stiffness terms K_{xy} and K_{yx} are treated as independent variables. Dr. Muszynska and I showed that these terms are not independent of one another and are properly characterized by the term $D\lambda\Omega$, where D is damping, λ (lambda) is the fluid circumferential average velocity ratio, and Ω is shaft rotative speed. We refer to this term, $D\lambda\Omega$, as the *tangential* or *quadrature* term. It is equivalent to the K_{xy} and K_{yx} terms, but our nomenclature enjoys the significant advantage that it defines stiffness in terms of the basic rotor dynamic parameters of damping, rotative speed, and lambda. This makes it a more useful model for diagnostic understanding.

“Cross stiffness” is sometimes represented as springs at 45Þ left and right below a rotor. This is totally incorrect. Cross stiffness is actually a sideways effect – a tangential, or quadrature, term that acts perpendicular to the direction of rotor displacement. Sometimes, people refer to these “perpendicular” terms as being composed of “real” and “imaginary” components; however, I have never liked using such nomenclature. There is nothing “imaginary” about the tangential term, and Bently Nevada always uses the word “quadrature” instead of “imaginary.” It can be easily observed in a rotor dynamic system. When you push down on a rotor, it moves not only down, but also to the side.

2. **Direct Stiffness Term**

Even the Direct Stiffness term has been badly misunderstood over the years because it is often modeled in terms of rotative speed under the assumption that only the circumferential profile of the supporting pressure wedge is of importance. Our work with externally pressurized bearings shows that Direct Stiffness is the result of the converging / diverging pressure wedge that is *axial* (along the rotor) in orientation, rather than *circumferential* (around the rotor). This clarifies a concept that has been misunderstood and misapplied for more than a century.

3. **Fluid Inertia Term**

Our rotor dynamic model introduced a term to quantify the fluid inertia effect. This effect can become very large and exhibit influence on the rotor that is quite significant, adding to the inertial effects of the rotor mass itself. However, although it behaves like inertia, and our model characterizes the rotor behavior quite accurately, it is incorrect to think of our term as a literal mass-based inertia. This can be observed very easily by introducing an air bubble into a cylindrical bearing system. The fluid inertia effect disappears instantly, and the system reverts to normal spring, mass, and damping characteristics. Therefore, it is perfectly clear that it is neither a Coriolis nor gyroscopic[1] effect. This fluid inertia effect acts very much like a negative spring and introduces a negative stiffness effect.

The before-mentioned improvements to the defining equations are necessary for the most accurate modeling and diagnostics of modern rotor dynamic systems. They also more clearly describe both simple and complex rotor systems. Finally, in addition to using the most up-to-date equations as the basis for calculations, we strongly advocate the use of a graphical analysis method known as Root-Locus in studying the stability of rotor dynamic systems. It has been very widely applied to study the stability of electronic control systems since its introduction by Walter Evans in 1954. However, it is less commonly applied to rotor dynamic systems, which is a shame since it is such a powerful analysis tool, vastly superior to older methods such as Campbell diagrams or logarithmic-decrement plots. Root-Locus is addressed very capably in the following excellent texts:

1. Evans, W. R., *Control-System Dynamics*, McGraw-Hill Book Company, Inc., New York, New York, 1954.

2. Nise, Norman S., *Control Systems Engineering with MATLAB*, Third Edition, John Wiley & Sons, 2000.

3. Ogata, Katsuhiko, *Modern Control Engineering*, Third Edition, Prentice-Hall, Inc., 1997.

If you would like further assistance with any of the concepts or methods covered in this summary, I would be pleased to help.

[1] It should be pointed out, however, that while fluid inertia is clearly not a gyroscopic effect, the existing descriptions of gyroscopic behavior are correctly done. Therefore, gyroscopics continue to be a valuable part of properly characterizing a rotor's behavior.